

# Oblique Propagation of Groundwaves Across a Coastline— Part III

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This paper, which is a continuation of two earlier papers of the same title, contains numerical results for the field anomaly near a coastline when the surface impedance changes in a linear manner between land and sea. The earlier results for an abrupt boundary are recovered as the width of the transition region is reduced to zero. In general, it is found that the characteristics of the transition region will not produce significant modifications of the transmitted field. However, the magnitude of the reflected field is greatly reduced as the width of the transition zone is increased beyond about one-quarter wavelength.

## 1. Introduction

In part I [Wait, 1963] of a series of papers of this title, the propagation of radio waves across a flat-lying coastline was investigated theoretically. In part II [Wait and Jackson, 1963], the influence of an elevation change between land and sea was considered. It is the purpose of the present paper to consider again the flat-lying coastline but with special attention given to a nonabrupt variation of the conductivity at the coastline.

As indicated in part I, the assumption of a sudden change of electrical properties gives rise to an apparent singularity of the field right at the coastline. It is shown in what follows that the "singularity" is not present when the surface impedance changes gradually at the junction or coastline.

## 2. Formulation

The model is the same as that used in part I. The essential features are described briefly here. The transmitting antenna, A, located on the land, is at a great distance from the coastline. For present purposes, it is only necessary to assume that A is a source of vertically polarized groundwaves. The receiving antenna, B, is relatively near the coastline but it may be on the sea. It is further assumed that B is equivalent to an infinitesimal vertical dipole and thus it responds only to the vertical electric field.

From consideration of reciprocity, it is clear that the role of transmitter and receiver may be interchanged. Thus, in general, it is most meaningful to express the results in terms of mutual impedance  $z_m + \Delta z_m$  between the respective terminal pairs of antennas A and B. Here  $z_m$  is the mutual impedance if the surface impedance were a constant  $Z$  for all points of the earth's surface. Thus,  $\Delta z_m$  is the modification of the mutual impedance which results from the presence of the inhomogeneity of surface impedance  $Z'$ . In the present problem, the surface

impedance of the land is  $Z$  while that of the sea is  $Z'$ . The situation is illustrated in figure 1 where the earth's surface is the  $(x, y)$  plane of a Cartesian coordinate system and the coastline is at  $x=0$ .

## 3. Statement of Formulas

Quoting from part I, the working formula for the mutual impedance increment  $\Delta z_m$  is given by

$$\frac{\Delta z_m}{z_m} \simeq -\frac{i}{2} e^{ikC_1 d_1} \int_{-\epsilon}^{\infty} \left[ C_1 \frac{df(x)}{dx} - ikf(x) \right] e^{-ikC_1 x} \times H_0^{(2)}[kC_1|x-d_1|] dx, \quad (1)$$

where

$$\begin{aligned} f(x) &= (Z' - Z)/\eta_0, \quad \eta_0 = 120\pi, \\ H_0^{(2)} &\text{ is the Hankel function of order zero of the second kind,} \\ C_1 &= \cos \theta_0, \quad k = 2\pi/\text{wavelength}, \\ d_1 &= \text{shortest distance from coastline to B (positive if B is over sea and negative if B is over land),} \end{aligned}$$

and, finally,  $\epsilon$  is chosen to be sufficiently large that  $f(x) \simeq 0$  for  $x < -\epsilon$ .

For purposes of the present paper, some further simplifications may be made. Specifically, it is noted that

$$f(x) = -\Delta_0(x) e^{i\pi/4}, \quad (2)$$

where  $\Delta_0(x)$  is approximately a real positive quantity whose magnitude is small compared with unity (e.g.,  $\Delta_0 < 0.1$ ). For example, at sufficiently large positive values of  $x$ ,

$$\begin{aligned} \Delta_0(x) \simeq \Delta_0 \simeq & \left( \frac{\epsilon_0 \omega}{\sigma_g + i\epsilon_g \omega} \right)^{1/2} \left( 1 - \frac{i\epsilon_0 \omega}{\sigma_g + i\epsilon_g \omega} \right)^{1/2} \\ & - \left( \frac{\epsilon_0 \omega}{\sigma' + i\epsilon' \omega} \right)^{1/2} \left( 1 - \frac{i\epsilon_0 \omega}{\sigma' + i\epsilon' \omega} \right)^{1/2}, \quad (3) \end{aligned}$$

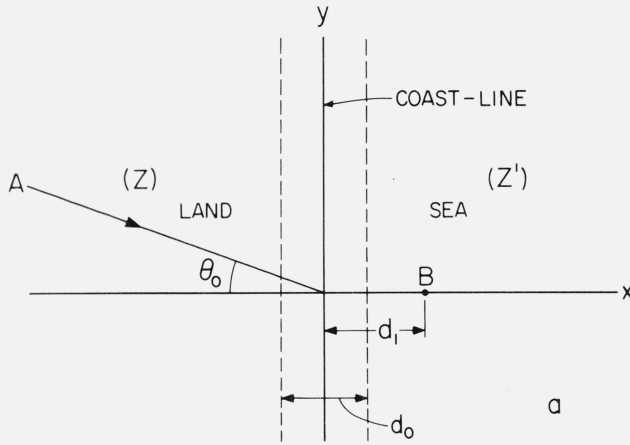


FIGURE 1a. Plan view of the earth's surface illustrating a coastline with a finite transition zone between land and sea.

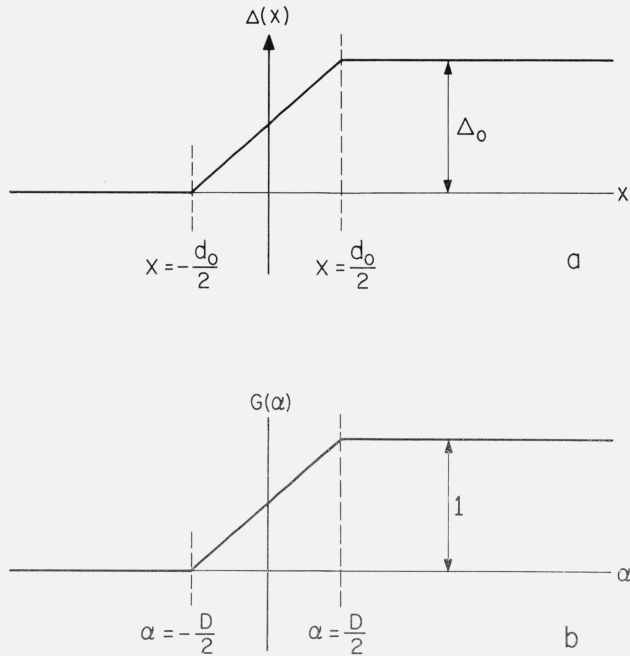


FIGURE 1b, c. The linear surface impedance variation.

where  $\sigma_g$  and  $\epsilon_g$  are the conductivity and permittivity of the land and  $\sigma'$  and  $\epsilon'$  are the conductivity and permittivity of the sea. In most cases of practical interest,  $\omega$  is sufficiently low and  $\sigma'$  is sufficiently large that

$$\Delta_0 \cong (\epsilon_0 \omega / \sigma_g)^{1/2}, \quad (4)$$

which is real.

#### 4. Linear Variation of Surface Impedance

In part I, it was assumed that  $\Delta_0(x)$  was a step function at  $x=0$  corresponding to an abrupt boundary. Here it is desirable to allow  $\Delta_0(x)$  to have a

finite transition region of width  $d_0$  where  $\Delta_0(x)$  varies in a linear manner. Thus

$$\begin{aligned} \Delta_0(x) &= 0 \text{ for } x < -d_0/2, \\ &= \left[ \frac{x + (d_0/2)}{d_0} \right] \Delta_0 \text{ for } -d_0/2 < x < d_0/2, \\ &= \Delta_0 \text{ for } x > d_0/2. \end{aligned} \quad (5)$$

With this model, the surface impedance is imagined to change in a linear manner over the distance  $d_0$  as indicated in figure 1b.

To simplify the computational problem, certain dimensionless quantities are introduced as follows.

$$\alpha = kC_1x, \quad \alpha_1 = kC_1d_1, \quad D = kC_1d_0,$$

and  $D_0 = D/C_1 = kd_0$ . Then, it easily follows that (1) may be written in the form

$$\frac{\Delta z_m}{z_m} = \frac{\Delta_0 e^{i3\pi/4}}{2} e^{i\alpha_1} \int_{-\infty}^{+\infty} H_0^{(2)}[|\alpha - \alpha_1|] e^{-i\alpha} \left[ C_1 \frac{dG}{d\alpha} - i \frac{G}{C_1} \right] d\alpha, \quad (6)$$

where, in the case of a linear variation,

$$\begin{aligned} G &= 0 \text{ for } \alpha < -D/2, \\ &= \frac{\alpha + (D/2)}{D} \text{ for } -D/2 < \alpha < D/2, \\ &= 1 \text{ for } \alpha > D/2. \end{aligned} \quad (7)$$

Therefore,

$$\begin{aligned} \frac{dG}{d\alpha} &= \frac{1}{D} \text{ for } -D/2 < \alpha < D/2, \\ &= 0 \text{ for } \alpha < -D/2 \text{ and } \alpha > D/2, \end{aligned} \quad (8)$$

as indicated in figure 1c where  $G$  is plotted versus  $\alpha$ . The mutual impedance formula now becomes

$$\frac{\Delta z_m}{z_m} = \Delta_0 \left[ C_1 Q_1 + \frac{Q_2}{C_1} \right] = \Delta_0 Q, \quad (9)$$

where

$$Q_1 = \frac{e^{i3\pi/4}}{2D} e^{i\alpha_1} \int_{-D/2}^{D/2} H_0^{(2)}[|\alpha - \alpha_1|] e^{-i\alpha} d\alpha, \quad (10)$$

and

$$\begin{aligned} Q_2 &= \frac{e^{i\pi/4}}{2} e^{i\alpha_1} \left\{ \int_{-D/2}^{D/2} \frac{\alpha + D/2}{D} H_0^{(2)}[|\alpha - \alpha_1|] e^{-i\alpha} d\alpha \right. \\ &\quad \left. + \int_{D/2}^{\infty} H_0^{(2)}[|\alpha - \alpha_1|] e^{-i\alpha} d\alpha \right\}. \end{aligned} \quad (11)$$

To effect these integrations, it is desirable to make use of the following two identities:

$$\frac{d}{dx} [xe^{\pm ix} [H_0^{(2)}(x) \mp iH_1^{(2)}(x)]] = e^{\pm ix} H_0^{(2)}(x), \quad (12)$$

and

$$\frac{d}{dx} \left[ \frac{xe^{\pm ix}}{3} [xH_0^{(2)}(x) + (1 \mp ix)H_1^{(2)}(x)] \right] = xe^{\pm ix} H_0^{(2)}(x), \quad (13)$$

where upper (or lower) signs are to be considered together. In verifying these identities, it is necessary to employ two well-known relations in Bessel function theory [MacLachlan, 1934]. These are

$$\frac{d}{dx} H_0^{(2)}(x) = -H_1^{(2)}(x), \quad (14)$$

and

$$\frac{d}{dx} H_1^{(2)}(x) = H_0^{(2)}(x) - \frac{H_1^{(2)}(x)}{x}. \quad (15)$$

It is now apparent that  $\Delta z_m/z_m$  may be expressed entirely in terms of Hankel functions of order zero and one, with various real arguments. Using such expressions, curves of the real and imaginary parts of  $Q$  have been prepared. Some of these are shown in figures 2a to 8b where, in each case, the abscissa is the parameter  $\alpha_1$  or  $kC_1 d_1$ .

For purposes of discussion, it is convenient to describe  $Q$  as the field anomaly resulting from the inhomogeneity. In view of the relation

$$\frac{\Delta z_m}{z_m} = \Delta_0 [\text{Re } Q + i \text{Im } Q], \quad (16)$$

it is evident, for  $\Delta_0$  real, that  $\text{Re } Q$  is a measure of the amplitude change of the field where  $\text{Im } Q$  is a measure of the phase. While the results are still valid if  $\Delta_0$  is complex, the above simple interpretation is not applicable. Also, it should be remembered that under all conditions,  $|\Delta z_m/z_m| \ll 1$  or  $|\Delta_0 Q| \ll 1$ , if the present results are to be given any confidence.

## 5. Discussion of Numerical Results

The curves in figures 2a and 2b are applicable to normal incidence. The sinusoidal-like ripples for negative values of  $\alpha_1$  may be regarded as an interference pattern resulting from the combination of the incident wave and the reflected wave. It is apparent that, as the electrical width  $D_0$  of the transition zone is increased, the magnitude of the reflected wave is generally decreased. On the other hand, the characteristics of the transmitted wave, for large positive values of  $\alpha_1$ , are not appreciably modified. However, it may be observed that in the proximity of the coastline, the nature of the field is profoundly influenced by the width of the transition region. In general, the rapid variations of the field are smoothed out when the transition distance  $D_0$  is increased. As may be

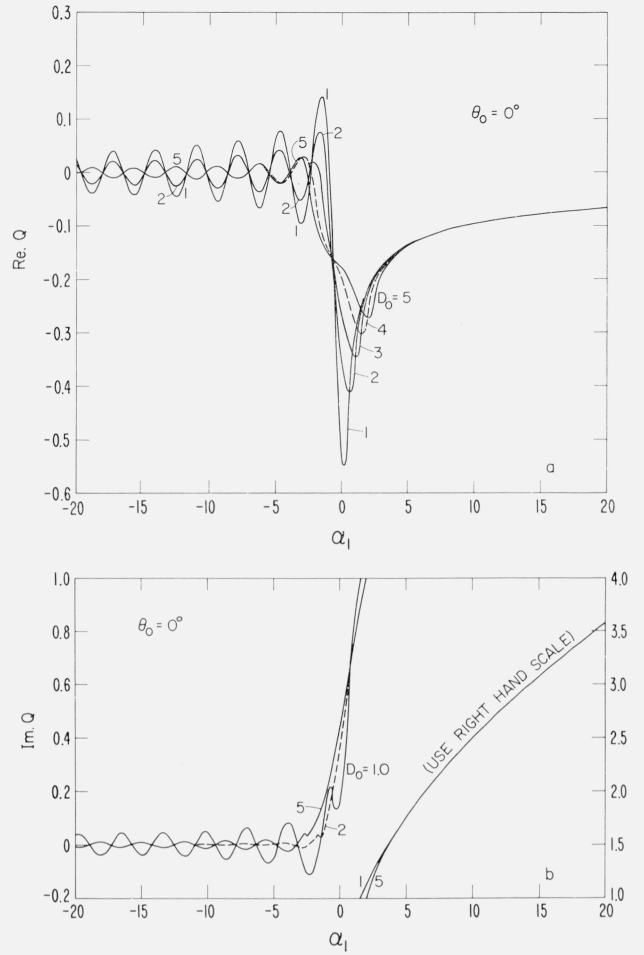


FIGURE 2. The real and the imaginary parts of  $Q$  as a function of  $\alpha_1$  or  $KC_1 d_1$  where  $d_1$  is the distance from the coastline.

seen in figure 1, the field is beginning to have a singular behavior for the smallest value of  $D_0$  (i.e., 1) which is very similar to the corresponding curves given in part I for the abrupt boundary (i.e.,  $D_0 = 0$ ).

The curves in figures 3a and 3b for  $\theta_0 = 20^\circ$  are very similar to those in figures 2a and 2b for  $\theta_0 = 0^\circ$ . Evidently, a slight obliquity in the traverse across the coastline does not modify noticeably the field behavior. However, for more oblique conditions, the curves in figures 4a, 4b, 5a, and 5b for  $\theta_0 = 40^\circ$  and  $50^\circ$  indicate that significant modifications take place for angles in this range. Principally, it is noted that when  $\theta_0$  is in the vicinity of  $45^\circ$  the reflected wave is greatly reduced in magnitude. This effect was also present in the case of an abrupt boundary.

The curves in figures 5a to 8b illustrate the behavior of the field anomaly for rather highly oblique angles. Again, as in the case of the abrupt boundary, the reflected wave is quite strong. Furthermore, the magnitude of the reflected wave is not appreciably reduced when  $D_0$  is increased from 1 to 5. However, in nearly all angles  $\theta_0$  the behavior of the transmitted field is not appreciably modified by changes in the width of the transition zone.

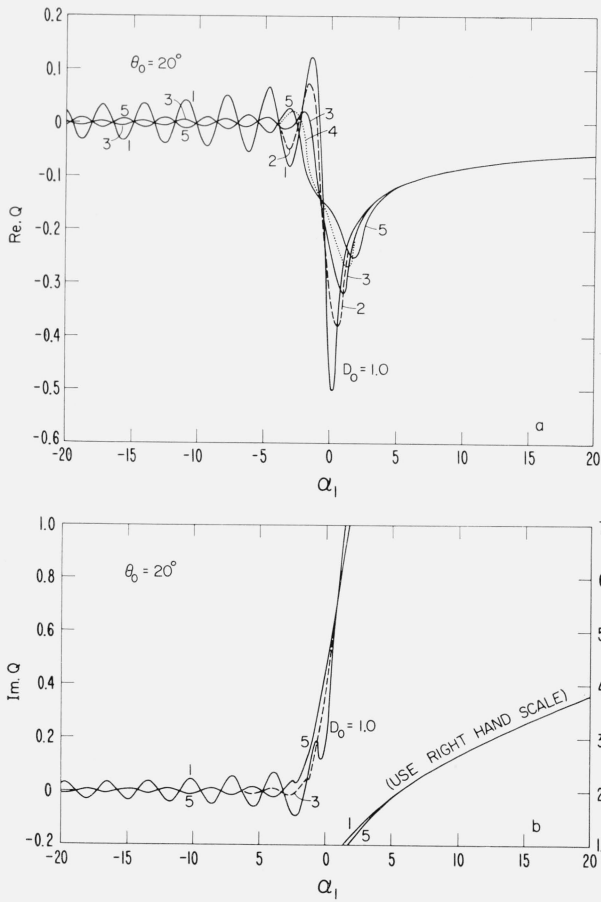


FIGURE 3. The real and the imaginary parts of  $Q$  as a function of  $\alpha_1$  or  $KC_1 d_1$  where  $d_1$  is the distance from the coastline.

The relative insensitivity of the  $Q$  versus  $a_1$  curves, at highly oblique angles, to variations in  $D_0$  is reminiscent of the reflection of waves from horizontally stratified media [Wait, 1962]. In the latter case it is known that the influence of diffusiveness or non-sharpness of the boundary is minimized at highly oblique incidence.

## 6. Conclusions

The results given in this paper would seem to shed considerable light on the nature of the vertical electric field near the boundary of separation. The singularly or infinite behavior of the field obtained in earlier studies, is not present when the surface impedance changes gradually between the land and sea portions. However, as the transition zone is diminished the field becomes rapidly varying in the vicinity of the coastline and becomes very similar to the predicted behavior for an abrupt boundary. A conclusion similar to this was arrived at by Godzinski [1962] whose analysis communicated to me in outline was restricted to normal incidence (i.e.,  $\theta_0 = 0^\circ$ ).

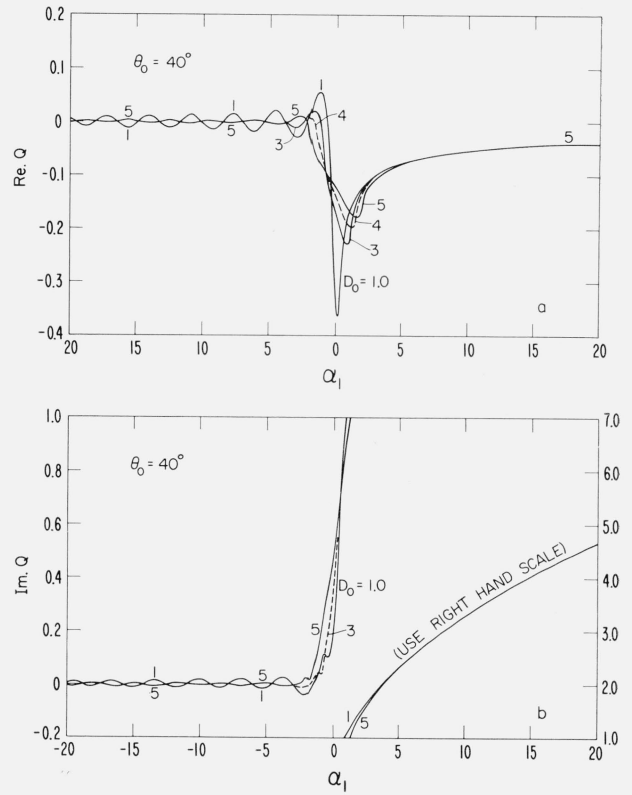


FIGURE 4. The real and the imaginary parts of  $Q$  as a function of  $\alpha_1$  or  $KC_1 d_1$  where  $d_1$  is the distance from the coastline.

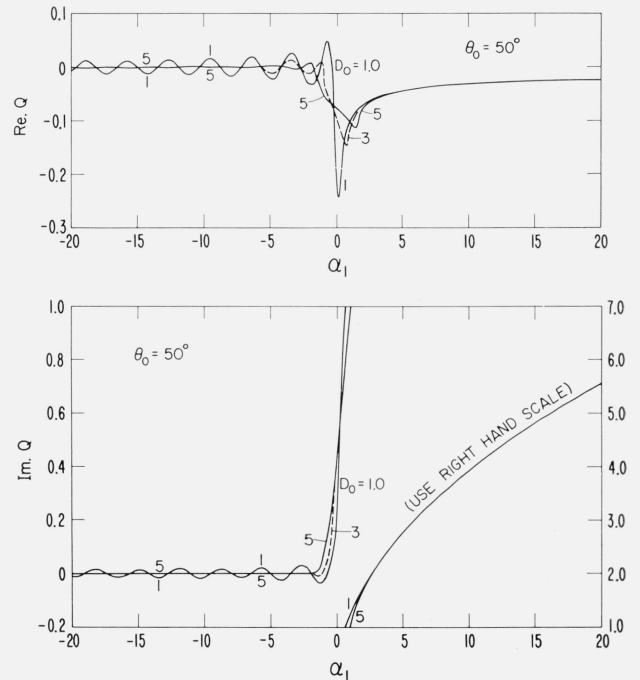


FIGURE 5. The real and the imaginary parts of  $Q$  as a function of  $\alpha_1$  or  $KC_1 d_1$  where  $d_1$  is the distance from the coastline.

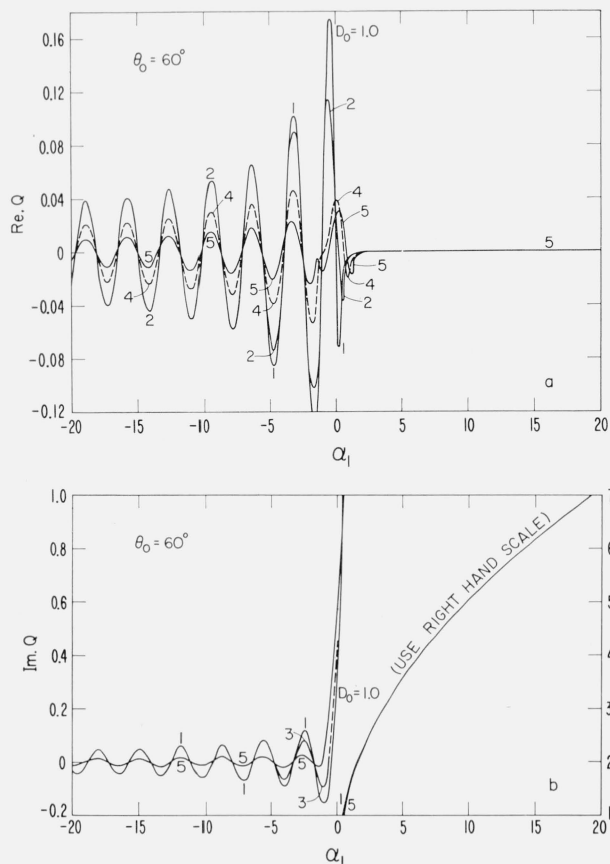


FIGURE 6. The real and the imaginary parts of  $Q$  as a function of  $\alpha_1$  or  $KC_1 d_1$  where  $d_1$  is the distance from the coastline.

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<sup>1</sup> The careful drafting of the figures is due to Jerry Hodges.

<sup>2</sup>  $v_z/c$  should be replaced by  $c/v_z$  in eqs (29) and (31),  $v_y/c$  should be replaced by  $c/v_y$  in eqs (30) and (32), and  $v_y/v_z$  should be replaced by  $v_z/v_y$  in eq (33).

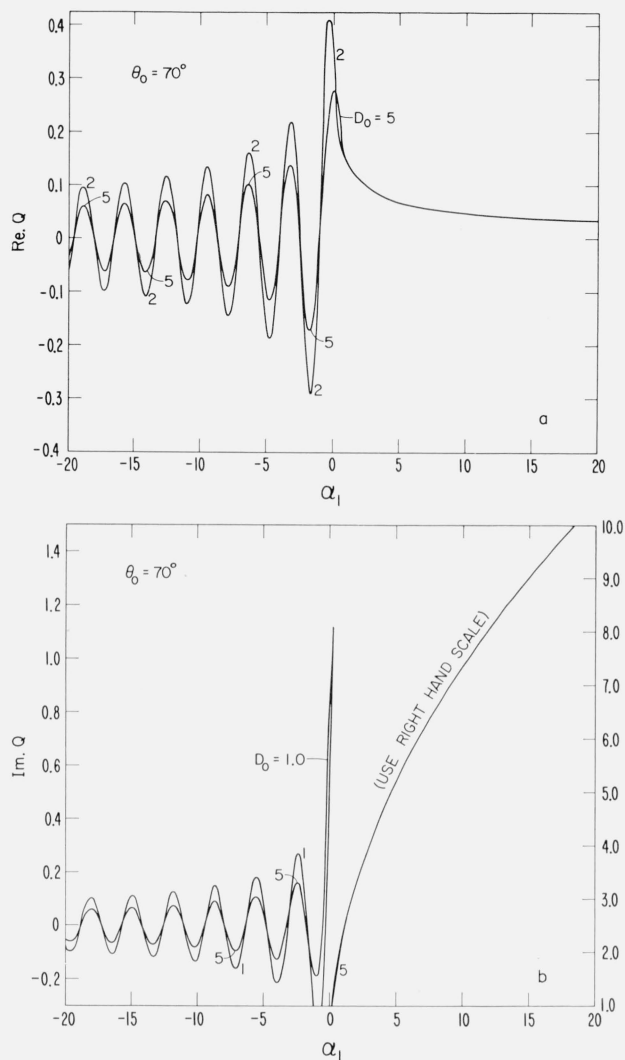


FIGURE 7. The real and the imaginary parts of  $Q$  as a function of  $\alpha_1$  or  $KC_1 d_1$  where  $d_1$  is the distance from the coastline.

## 7. References

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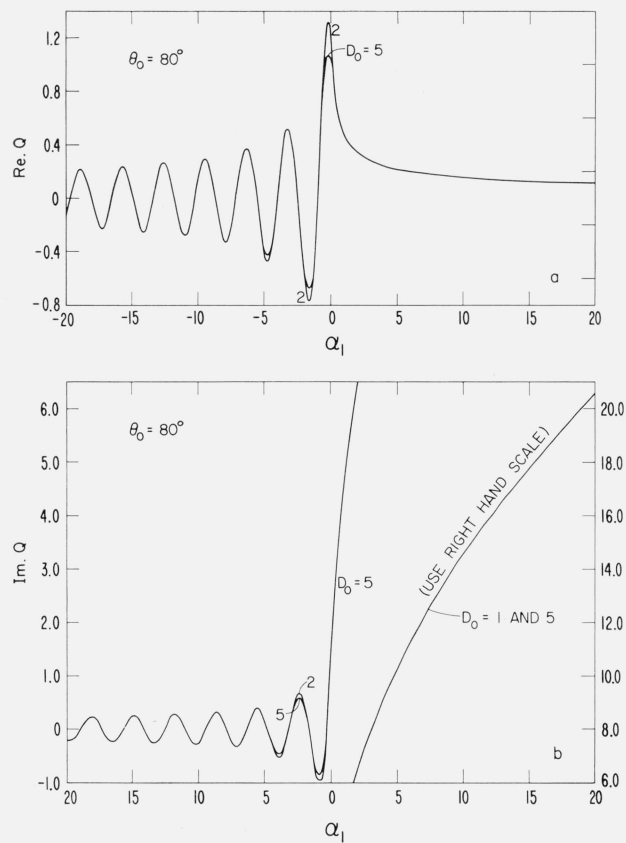


FIGURE 8. The real and the imaginary parts of  $Q$  as a function of  $\alpha_1$  or  $KC_1 d_1$  where  $d_1$  is the distance from the coastline.

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